Firm Heterogeneity:
Implications for Wage Inequality and Aggregate Growth

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Motivation

- Matched employer-employee data reveal that more productive firms are larger, pay higher wages, and are more likely to export.
- Research Questions
  1. What are the quantitative implications of firm productivity differences for aggregate productivity and economic growth?
  2. Are there large firm efficiency differences or large differences in input quality that account for these observations?

Evidence from Firm Micro Data

- U.S. Average total factor productivity (TFP) 90/10 ratio within manufacturing industries is 1.92… Syverson (2004)
- 90/10 ratio across all Danish firms for labor productivity is 2.3 and firm wages is 1.8… Bagger et al. (2010)
- 90/10 TFP ratios are over 5 for firms in both China and India… Hsieh & Klenow (2009)

Evidence from Firm Trade Data

- Few firms export and those that do are larger and more productive.
- Exporting firms pay wage premiums.
- Exporting firms are found in all industries in the manufacturing sector.
- Sources
  - US: Bernard and Jensen (1995, 1999)
  - France: Eaton et al. (2004, 2008)
  - Denmark: Pedersen (2009)

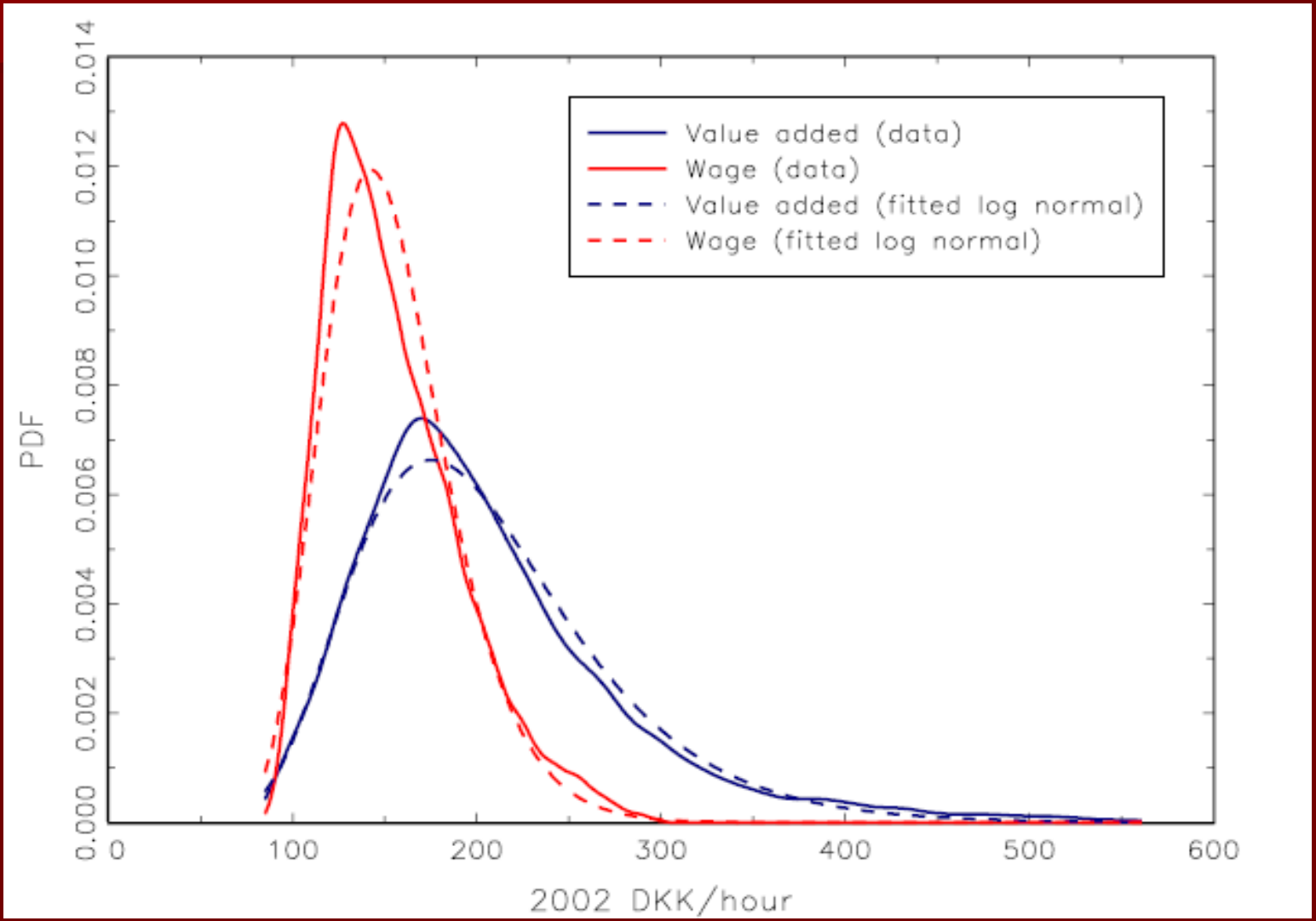
The Danish Project

- Lentz and Mortensen (2008), "An Empirical Model of Growth Through Product Innovation." *Econometrica*.
- **Lentz and Mortensen (2010),"Labor Market Models of Firm and Worker Heterogeneity." *Annual Review of Economics*.**
- Bagger, Christensen, and Mortensen (2010), "Wage and Productivity Dispersion: Labor Quality or Rent Sharing?," working paper.

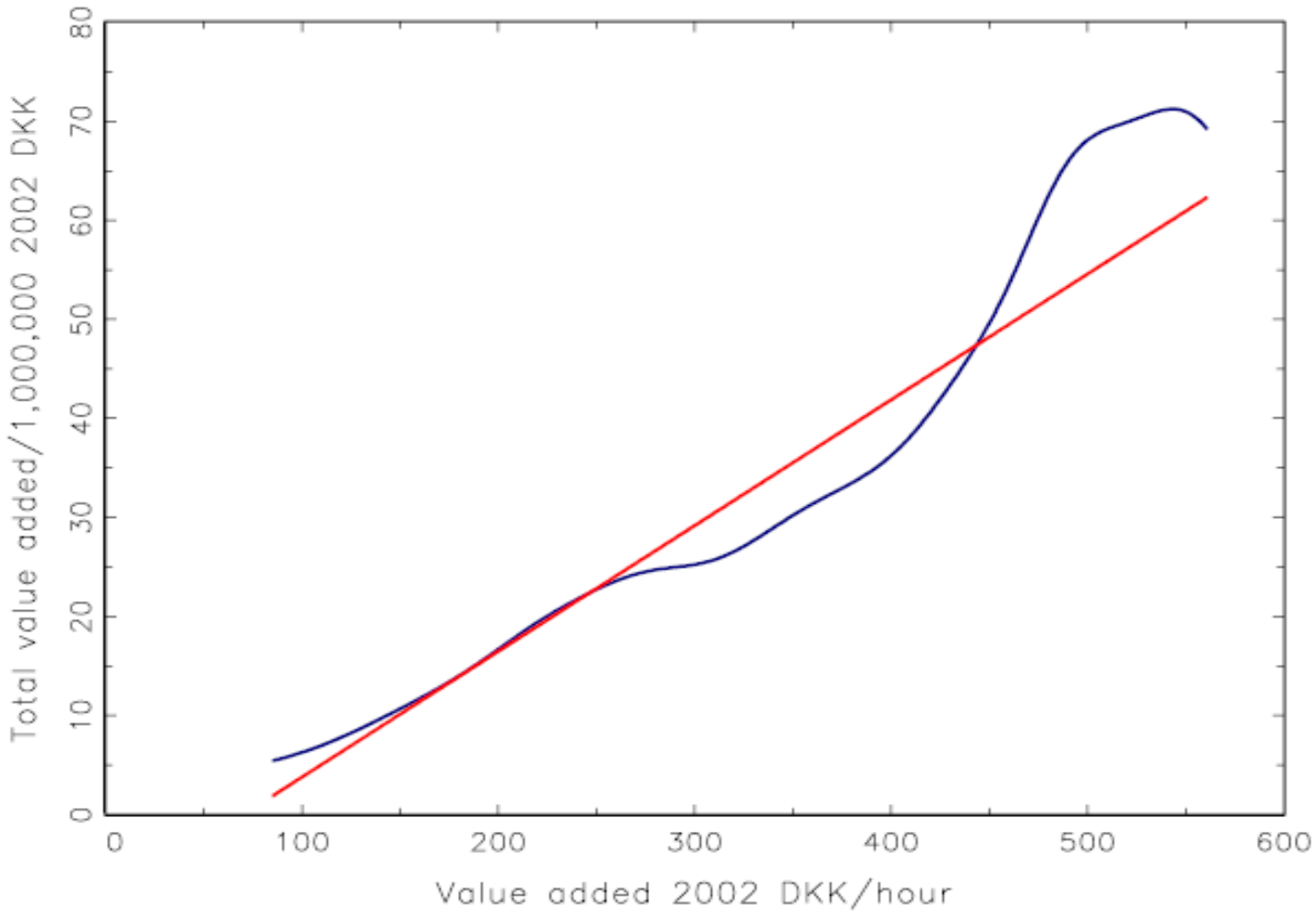
The Danish Data

- IDA: Contains worker identity, earning, and characteristics (age, gender, education, working experience, labor market histories, household structure) and employer IDs.
- Stratified rolling panel firm survey: Contains firm accounting information (sales, materials purchases, investment, capital book value, exports and imports) for each year.
- These are linked to create an matched employer-employee panel which is available for research at Aarhus University.

Firm Productivity and Wage Distributions: All Private Danish Firms



Firm Size vs Productivity



Growth Through Innovation I

Grossman and Helpman (1991)
Klette and Kortum (2004)

- Final goods and services are produced using many differentiated intermediate goods and services as inputs. (Dixit-Stiglitz)
- Intermediate products are produced with labor and capital.
- More productive versions of each intermediate product type are created from time to time that replace older versions.
- The cost of product innovation (R&D) rises at the margin.

Growth Through Innnovation II

- New firms enter and continuing firms grow by creating new products and shrink and exit through product destruction.
- Firms differ with respect to the expected productivity of the intermediate goods and services that they create.
- The supplier of the current version of each product sets its price.

Empirical Implications I

- Firms that create higher quality intermediate products are more profitable, invest more in R&D, and supply a growing fraction of the products in each entry cohort.
- As new products and services displace old, this process of creative-destruction induces reallocate of workers from the less to the more productive firms.

Empirical Implications II

- Aggregate growth is determined by firm innovation rates and the speed with which resources are reallocated to the most innovative firms.
- Steady state firm heterogeneity in productivity is sustained by the cost of innovation, initial uncertainty about type, and the process of creative-destruction.

Parameter Estimation I

- Create a structural model of firm interactions that generates the relationship between firm R&D investment decisions and observables.
- Using the equilibrium steady state relationships of the model, the parameters are estimated by the method of simulated moments using observables drawn for a panel of Danish firms over the period 1992-1997.

Table II: Data moments (standard error in parenthesis)

	1992	1997		1992	1996
Survivors	4872.000 –	3628.000 (32.132)	$\mathrm{Cor}[\frac{Y}{N^*}, \frac{Y_{+1}}{N^*_{+1}}]$	0.476 (0.088)	0.550 (0.091)
$\mathrm{E}[Y]$	26277.262 (747.001)	31860.850 (1031.252)	$\mathrm{Cor}[\frac{Y}{N^*}, \Delta\frac{Y}{N^*}]$	−0.227 (0.103)	−0.193 (0.057)
$\mathrm{Med}[Y]$	13472.812 (211.851)	16448.965 (329.417)	$\mathrm{Cor}[\frac{Y}{N^*}, \frac{\Delta Y}{Y}]$	−0.120 (0.016)	
$\mathrm{Std}[Y]$	52793.105 (5663.047)	64120.233 (7741.448)	$\mathrm{Cor}[\frac{Y}{N^*}, \frac{\Delta N^*}{N^*}]$	0.119 (0.032)	
$\mathrm{E}[W]$	13294.479 (457.466)	15705.087 (609.595)	$\mathrm{E}[\frac{\Delta Y}{Y}]$	−0.029 (0.008)	
$\mathrm{Med}[W]$	7231.812 (92.720)	8671.939 (154.767)	$\mathrm{Std}[\frac{\Delta Y}{Y}]$	0.550 (0.067)	
$\mathrm{Std}[W]$	30613.801 (6750.399)	35555.701 (8137.541)	$\mathrm{Cor}[\frac{\Delta Y}{Y}, Y]$	−0.061 (0.012)	
$\mathrm{E}[\frac{Y}{N^*}]$	384.401 (2.907)	432.118 (5.103)	Within	1.015 (0.146)	
$\mathrm{Med}[\frac{Y}{N^*}]$	348.148 (1.829)	375.739 (2.139)	Between	0.453 (0.112)	
$\mathrm{Std}[\frac{Y}{N^*}]$	205.074 (19.633)	305.306 (42.491)	Cross	−0.551 (0.196)	
$\mathrm{Cor}[Y, W]$	0.852 (0.035)	0.857 (0.045)	Exit	0.084 (0.066)	
$\mathrm{Cor}[\frac{Y}{N^*}, N^*]$	−0.018 (0.013)	−0.026 (0.011)			
$\mathrm{Cor}[\frac{Y}{N^*}, Y]$	0.198 (0.036)	0.143 (0.038)			

Note: Unit of measurement is 1,000 DKK. The BHC growth decomposition moments are stated as fractions of total growth.

TABLE III

PARAMETER ESTIMATES (STANDARD ERROR IN PARENTHESES)[a]

			$\tau=1$	$\tau=2$	$\tau=3$
c_0/Z	175.8159	ϕ_τ	0.8478	0.0952	0.0570
	(12.9903)		(0.0133)	(0.0239)	(0.0230)
c_1	3.7281	ϕ^*_τ	0.7387	0.1614	0.0999
	(0.0411)		(0.0165)	(0.0403)	(0.0394)
κ	150.0126	ξ_τ	0.0000	0.3524	0.4168
	(2.0581)		(0.0000)	(0.0471)	(0.0750)
Z	16,859.4212	β_q	0.4275	0.4275	0.4275
	(329.3350)		(0.0199)	(0.0199)	(0.0199)
β_Z	0.9577	γ_τ	0.0000	0.0553	0.0566
	(0.0227)		(0.0000)	(0.0017)	(0.0016)
o_Z	608.2725	K_τ	0.5408	0.2776	0.1816
	(113.1480)		(0.0170)	(0.0703)	(0.0700)
η	0.0451	ν_τ	0.0000	3.2392	3.5332
	(0.0016)		(0.0000)	(0.1779)	(0.2540)
σ^2_Y	0.0323	$\bar{\pi}_\tau(\sigma)$	0.0000	0.2499	0.2690
	(0.0037)		(0.0000)	(0.0121)	(0.0166)
σ^2_W	0.0254	$\mathrm{Med}[\tilde{q}_\tau]$	1.0000	1.1495	1.1769
	(0.0000)		(0.0000)	(0.0221)	(0.0339)
δ	0.0707	$E[\tilde{k}_\tau]$	1.0000	2.3503	2.4833
	(0.0019)		(0.0000)	(0.1062)	(0.1470)
μ	1.3406				
	(0.0519)				
ℓ	45.7344				
	(0.8958)				
g	0.0139				
	(0.0006)				

[a]Equilibrium wage is estimated at $w = 190.239$. Estimation is performed for given $\sigma = 1$.

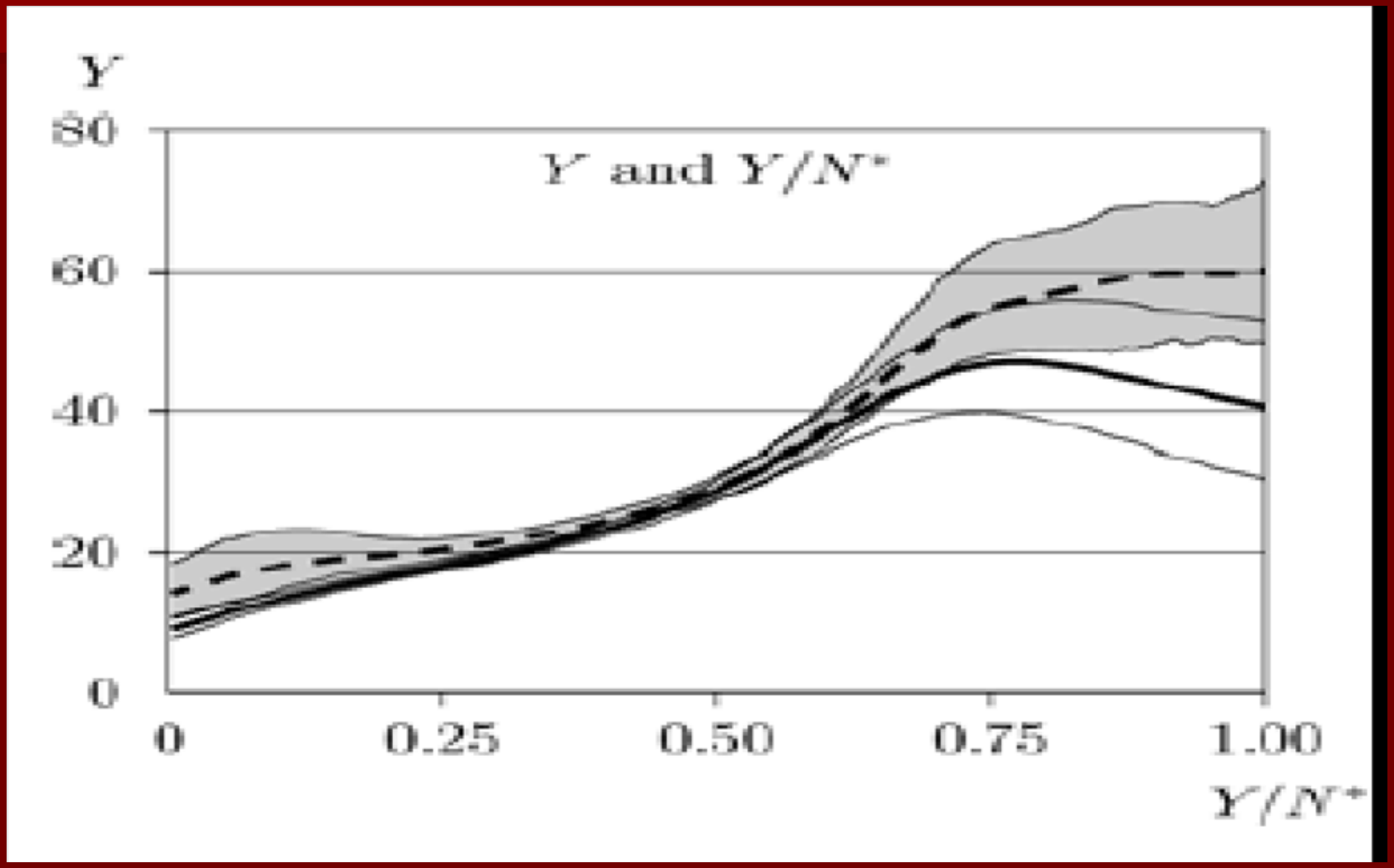
Model Fit

TABLE IV

MODEL FIT (DATA IN TOP ROW, ESTIMATED MODEL IN BOTTOM ROW)

	1992	1997		1992	1996
Survivors	4872.000	3628.000	$\mathrm{Cor}[\frac{Y}{N^*}, \frac{Y_{+1}}{N^*_{+1}}]$	0.476	0.550
	4872.000	3604.315		0.716	0.718
$E[Y]$	26,277.262	31,860.850	$\mathrm{Cor}[\frac{Y}{N^*}, \Delta\frac{Y}{N^*}]$	−0.227	−0.193
	23,023.834	27,252.981		−0.342	−0.352
$\mathrm{Med}[Y]$	13,472.812	16,448.965	$\mathrm{Cor}[\frac{Y}{N^*}, \frac{\Delta Y}{Y}]$	−0.120	
	13,352.797	15,382.112		−0.094	
$\mathrm{Std}[Y]$	52,793.105	64,120.233	$\mathrm{Cor}[\frac{Y}{N^*}, \frac{\Delta N^*}{N^*}]$	0.119	
	31,013.660	37,224.359		0.123	
$E[W]$	13,294.479	15,705.087	$E[\frac{\Delta Y}{Y}]$	−0.029	
	11,772.341	13,735.062		0.024	
$\mathrm{Med}[W]$	7231.812	8671.939	$\mathrm{Std}[\frac{\Delta Y}{Y}]$	0.550	
	7122.721	8154.757		0.771	
$\mathrm{Std}[W]$	30,613.801	35,555.701	$\mathrm{Cor}[\frac{\Delta Y}{Y}, Y]$	−0.061	
	14,716.584	17,431.300		−0.042	
$E[\frac{Y}{N^*}]$	384.401	432.118	Within	1.015	
	379.930	417.041		0.969	
$\mathrm{Med}[\frac{Y}{N^*}]$	348.148	375.739	Between	0.453	
	346.456	378.623		0.364	
$\mathrm{Std}[\frac{Y}{N^*}]$	205.074	305.306	Cross	−0.551	
	202.134	223.173		−0.446	
$\mathrm{Cor}[Y, W]$	0.852	0.857	Exit	0.084	
	0.927	0.928		0.113	
$\mathrm{Cor}[\frac{Y}{N^*}, N^*]$	−0.018	−0.026			
	−0.031	−0.024			
$\mathrm{Cor}[\frac{Y}{N^*}, Y]$	0.198	0.143			
	0.170	0.176			

Fit of Size vs Productivity



Fit of Productivity Distribution

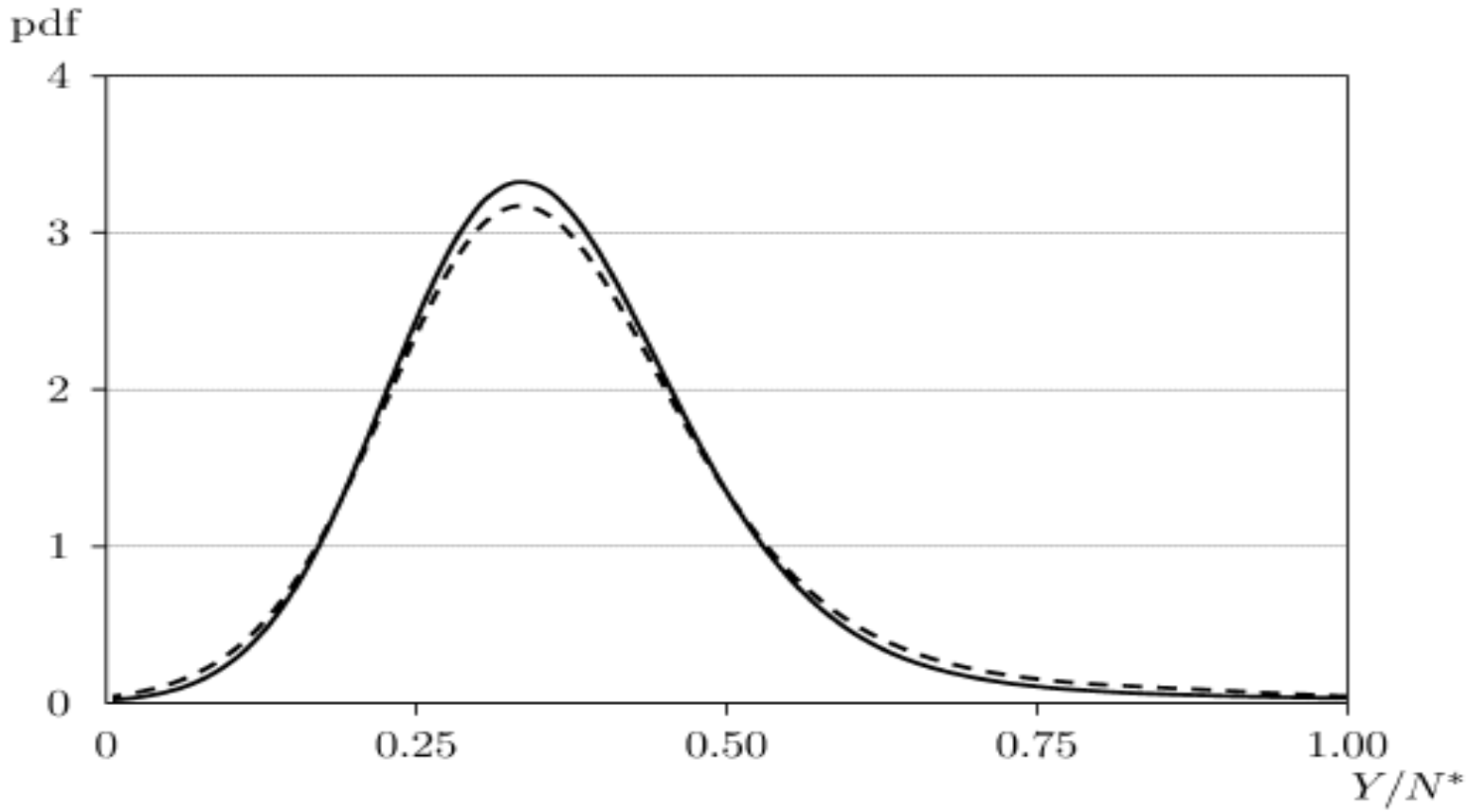


FIGURE 3.—Firm productivity distribution fit, 1992. Value added (Y) measured in 1 million DKK. Labor force size (N^*) measured in efficiency units. Estimated model is solid; data are dashed.

Decomposing Productivity Growth

Growth rate in output per worker =
Entry/Exit + Selection + Residual

- Entry/Exit: Attributable to the fact that new entrants are more productive than firms that exist.
- Selection: Attributable to the fact that more productive firms in each entry cohort become larger over time.
- Residual: Counterfactual contribution to growth of continuing firms in the absence of selection and entry and exit.

Model Based Productivity Growth Decomposition

Table VI: The productivity growth rate and its components (std error in parenthesis)

	$\sigma = 1.000$	$\sigma = 0.500$	$\sigma = 0.750$	$\sigma = 1.250$
g	0.0139	0.0123	0.0131	0.0144
	(0.0006)	—	—	—
Decomposition (fraction of g)				
– Entry/exit	0.2110	0.1876	0.1959	0.2498
	(0.0149)	—	—	—
– Residual	0.2608	0.2316	0.2425	0.2788
	(0.0128)	—	—	—
– Selection	0.5282	0.5809	0.5615	0.4715
	(0.0270)	—	—	—

Note: Standard deviation estimates obtained by bootstrap. Standard errors have not been calculated for the robustness analysis.

Empirical Labor Productivity Growth Decomposition

The decomposition takes the form[8]

$$\Delta\Theta_t = \sum_{i\in\hat{C}_t} s_{it-1}\Delta\theta_{it} + \sum_{i\in\hat{C}_t} \theta_{it-1}\Delta s_{it} + \sum_{i\in\hat{C}_t} \Delta\theta_{it}\Delta s_{it} + \sum_{i\in\hat{E}_t} \theta_{it}s_{it} - \sum_{i\in\hat{X}_t} \theta_{it-1}s_{it-1}, \tag{32}$$

where $\Theta_t = \sum_i s_{it}\theta_{it}$, $\theta_{it} = Y_{it}/N_{it}$, and $s_{it} = N_{it}/N_t$; $\hat{C}_t$ is the set of continuing firms, $\hat{E}_t$ is the set of entering firms and $\hat{X}_t$ is the set of exiting firms.

References: Foster, Haltiwanger and Krizan (2001) and Patrin and Levinsohn (2004).

Theoretical Implications

- Changes in employment shares for each firm type are stationary in the steady state.
- Hence, the "between" and "cross" terms are zero in a simple model.
- Hypothesis: If the economy is in a steady state, then none zero values in these terms of a FHK decomposition are "noise."

Analysis of FHK Decomposition

TABLE V

FHK GROWTH DECOMPOSITION AND COUNTERFACTUALS[a]

			Steady State With Entry			
	Data	Point Estimate	Point Estimate	Counter-factual 1	Counter-factual 2	Counter-factual 3
Within	1.0149	0.9676	1.1449	0.8807	0.9268	0.8053
Between	0.4525	0.3665	0.3134	0.0695	0.1124	0.0003
Cross	−0.5514	−0.4465	−0.6531	−0.1442	−0.2334	0.0003
Exit	0.0839	0.1124	0.1338	0.1321	0.1321	0.1321
Entry	—	—	0.0610	0.0618	0.0620	0.0620
Survivor growth rate	0.0165	0.0165	0.0164	0.0164	0.0164	0.0164
Growth rate	—	0.0139	0.0139	0.0139	0.0139	0.0139

[a]All counterfactuals are performed on the estimated model. Counterfactual 1 imposes zero measurement error, $\sigma_Y^2 = \sigma_W^2 = 0$. In addition to counterfactual 1, counterfactual 2 imposes zero demand realization dispersion while maintaining $E[\tilde{z}]$ at its estimated level, $\mathrm{Var}[\tilde{z}] = 0$. In addition to counterfactual 2, counterfactual 3 imposes zero within type quality improvement dispersion, $\tilde{q}_\tau = (1-\kappa)/(1-\kappa-\bar{\pi}_\tau)\ \forall\tau$. The growth rate is the annual growth rate in aggregate value added per worker calculated on existing firms at a given point in time. The survivor growth rate describes the value added per worker growth rate of surviving 1992 firms.

TFP Differences or Differences in Input Quality?

- Cobb-Douglas production function

$$\ln Y_{it} = \ln p_{jt} + \alpha_K \ln K_{jt} + \alpha_L \ln L_{jt} + \varepsilon_{it}$$

where

$$p_{jt} = \gamma_j + \rho p_{jt-1} + \varepsilon_{jt}$$

is TFP of firm j in year t,

K_{jt} is the capital input in year t,

$L_{jt}=\Sigma_{i\in Ijt} a_i$ is labor input

where a_i is the "ability" of worker I, and

I_{jt} is the set of worker in firm j at time t.

- AKM wage equation is

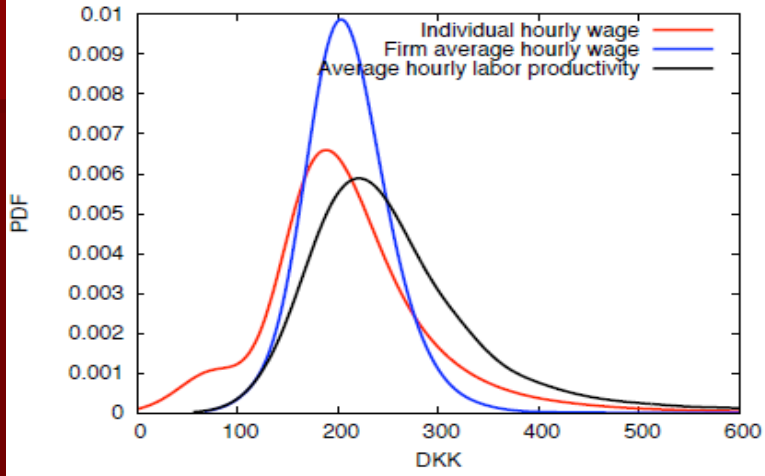
$$\ln w_{ijt} = \ln a_{it} + \ln \omega_{jt}$$

- The "price of ability" is determined by (Stole-Zwiebel) bargaining:

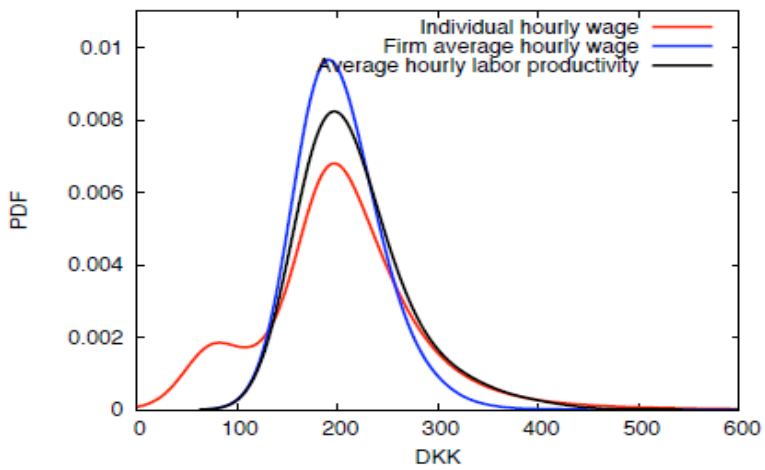
$$\omega_{jt} = (1-\beta)b + \alpha_L\beta/(1-\beta+\alpha_L\beta)Y_{jt}/L_{jt} + v_{jt}$$

where b represents "home production" and β denotes worker "bargaining power."

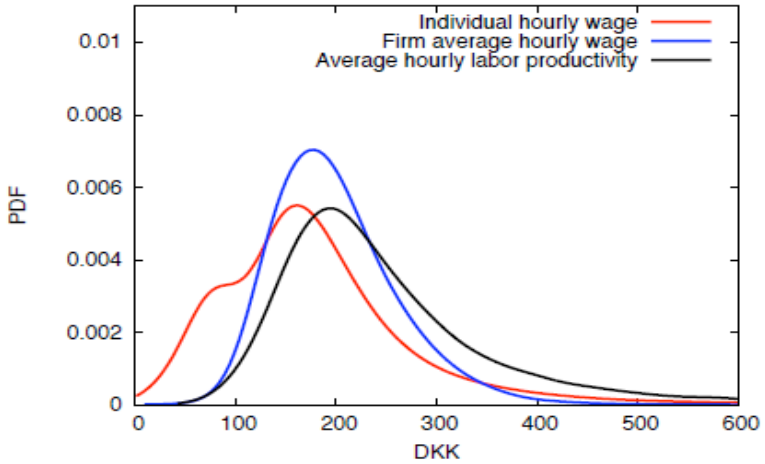
Dispersion by Industry



Manufacturing

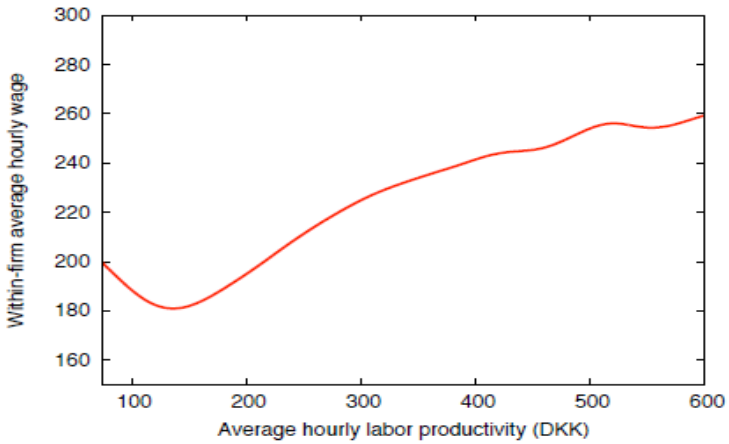


Construction

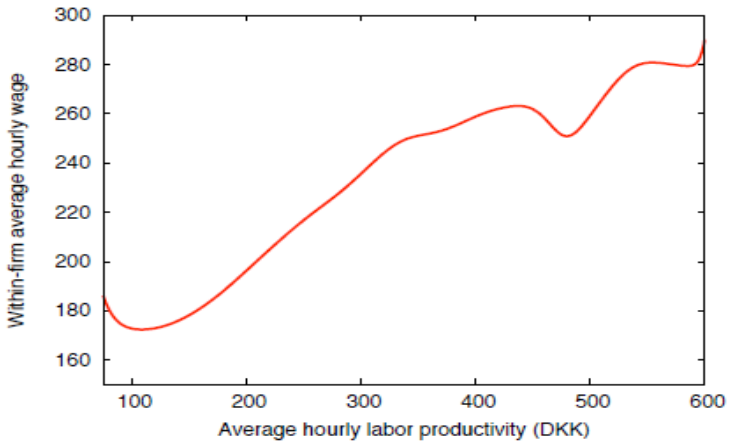


Wholesale & Retail

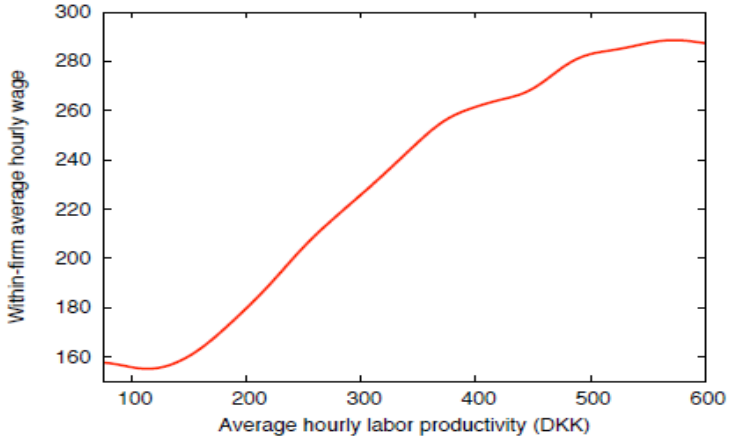
Wage-Productivity Correlation



Manufacturing



Construction



Wholesale & Retail

Model Estimation I

- Linked Danish manufacturing data: Annual observations for 1995-2005
  1. Contains worker identity, wage, experience and employer ID in November of each year for all firms.
  2. Stratified rolling panel firm survey: Contains firm value added, wage bill, book value, and capital purchases information for each year.

Model Estimation II

- Two Stage Procedure:
  1. Use IDA data and AKM decomposition to estimate a_{jt} for each i and t and use individual wage observations to identify ω_{jt} and L_{jt}.
  2. Under the assumption that TFP (p_{jt}) is an AR1 process, use firm data and estimates of ω_{jt} and L_{jt} to estimate wage bargaining and production function parameters.

Wage and Productivity Statistics

	Manufacturing	Construction	Wholesale & Retail
Hourly value added/FTE:			
Mean	257.60	217.74	246.69
S.d.	93.09	56.63	106.11
Firm specific average wage:			
Mean	198.34	190.51	177.35
S.d.	112.24	110.95	136.60
Cor. w/ hrly VA/FTE	0.46	0.47	
Individual wage:			
Mean	214.93	207.07	184.72
S.d.	125.59	111.08	133.81
Cor. w/ hrly VA/FTE	0.14	0.17	

Structural Parameter Estimates

	LS	Within	ACF
Output elasticity w.r.t. K, α_K	0.1146 (0.0018)	0.1072 (0.0033)	0.3263
Output elasticity w.r.t. L, α_L	0.8854 (0.0057)	0.8928 (0.0103)	0.6896
Workers' barganing power, β			0.4154
Workers' outside option, b			205.5841

Individual Wage Decomposition

	Manufacturing
$\mathrm{Var}[\ln w_{it}]$	0.2192
$\mathrm{Cov}[\ln a_{it}, \ln w_{it}]$	0.07994 (0.68730)
$\mathrm{Cov}[\ln w(K_{jt}, L_{jt}, p_{jt}), \ln w_{it}]$	0.01583 (0.13611)
$\mathrm{Cov}[\varepsilon^{W}_{jt}, \ln w_{it}]$	0.02054 (0.17659)

Labor Productivity Decomposition

$\mathrm{Var}\left[\ln \frac{Y_{jt}}{N_{jt}}\right]$	0.09685
$\mathrm{Cov}\left[\alpha_K \ln \frac{K_{jt}}{N_{jt}}, \ln \frac{Y_{jt}}{N_{jt}}\right]$	0.03967 (0.40977)
$\mathrm{Cov}\left[\alpha_L \ln \frac{L_{jt}}{N_{jt}}, \ln \frac{Y_{jt}}{N_{jt}}\right]$	0.00454 (0.04691)
$\mathrm{Cov}\left[(\alpha_L + \alpha_K - 1) \ln N_{jt}, \ln \frac{Y_{jt}}{N_{jt}}\right]$	0.00079 (0.00818)
$\mathrm{Cov}\left[p_{jt}, \ln \frac{Y_{jt}}{N_{jt}}\right]$	0.03748 (0.38701)
$\mathrm{Cov}\left[\varepsilon^Y_{jt}, \ln \frac{Y_{jt}}{N_{jt}}\right]$	0.01436 (0.14824)

Policy Issues I

- Wage Dispersion
  - Rent sharing is "unfair" in the sense that equally qualified workers are paid differently.
  - However, wage differentials motivate efficient reallocation. Christensen et. al (2005)

Results suggests that the efficient incentive effect is important.

Policy Issues II

- Efficiency: The Schumpeter tradeoff
  - Monopoly pricing distorts input decisions.
  - "Business stealing" externality adversely affects R&D investment.

Further research suggests that the latter is more important. Hence, existing patent and trade-mark protection may be of inadequate.