# Learning, Expectations, and Endogenous Business Cycles

Artur Doshchyn & Nicola Giommetti

I.S.E.O. Summer School

June 19, 2013

# Who we are

Two students writing a joint thesis to obtain MSc degree in Advanced Economics and Finance at Copenhagen Business School.

Artur Doshchyn

artur.doshchyn@gmail.com

What's next: Economic analyst at A.P. Moller-Maersk; PhD at some cool university :)

Nicola Giommetti

n.giommetti@email.com

What's next: PhD in Finance at University of Texas, Austin

# What is it all about

We show that business cycles can emerge and proliferate in the economy endogenously due to the way economic agents learn, form their expectations, and make decisions regarding savings and production for future periods. There are no exogenous shocks of any kind to productivity or any other fundamental parameter of the economy, in contrast to the Real Business Cycle (RBC) models.

# A bit of background

- **A. Pigou:** expectations may play a central role during the short-term fluctuations;
- **J.M. Keynes:** animal spirits;
- **Rational Expectations Revolution:** rapid development of sophisticated modelling in macroeconomics; however, very hard to generate *endogenous* business cycles in perfect foresight environment (done by Grandmont, 1985);
- **RBC (F. Kydland and E. Prescott):** external productivity shock;
- **Learning and expectations:** several papers investigating improvements in RBC models, most notably Cellarier, 2008; Eusepi and Preston, 2011.

# Our motivation and goal

- **Diverge from DSGE:** switch off the 'God mode' and inhabit our theoretical model with human beings;
- **No shocks:** demonstrate that economy may fluctuate even when there are no stochastic shocks;
- **KISS:** build a simple, compact and solvable macroeconomic model.

# Setup

- Growth-less economy with $N$ identical firms and $H$ identical households;
- Agents perceive themselves to be too small to affect economy;
- Homogenous output used both as consumer and capital good;
- Households provide their savings to firms to invest without nominal interest rate;
- Constant money stock $M$ with velocity 1, so that $Y_t P_t = M$;
- Each period one household disposes $M/H$ of cash, a sum of its labour income and savings from the previous period, implying:

$$M = HS_{t-1} + w_t NL_t. \tag{1}$$

# Timing of events within one period

Firms decide on production $Q_t$ and employment $L_t$ given $P_t^e$ and $K_t$; wage $w_t$ is determined

Price $P_t$ is determined

Agents observe $P_t$ and form expectation $P_{t+1}^e$

Households decide on savings $S_t$ and consume rest of income

Firms invest what households save

# Firms

- Firms have Cobb-Douglas production technology and use two factors: labour and capital.
- Each firms at the beginning of period $t$ solves:

$$\max_{L_t} \left\{ P_t^e A K_t^\alpha L_t^\beta - w_t L_t \right\}, \tag{2}$$

which yields demand for labour as a function of nominal wage.

- Combining with nominal constraint (1), we get actual employment and production. Total economy output is:

$$Y_t = \frac{M - HS_{t-1}}{\beta P_t^e}. \tag{3}$$

# Households

- Households use logarithmic utility function to value current real consumption and real savings expressed in expected purchasing power next period.
- Each household's problem in period $t$ is:

$$\max_{S_t} \left\{ \ln\left(\frac{I_t - S_t}{P_t}\right) + \delta \ln\left(\frac{S_t}{P^e_{t+1}} + C\right) \right\}. \tag{4}$$

- $C > 0$ is required to adjust marginal utility of savings down.
- Solving (4) yields savings decision:

$$S_t = \frac{\delta}{1+\delta} I_t - \frac{C}{1+\delta} P^e_{t+1}, \tag{5}$$

which increases in real interest rate $P_t / P^e_{t+1} - 1$.

# Actual law of motion

- Since $I_t = \frac{M}{H}$, it is straightforward to derive the *actual law of motion* (ALM) of price:

$$P_t = \frac{\beta M(1+\delta)}{M + HCP_t^e} P_t^e. \tag{6}$$

- Call $D(P_t^e) = \frac{\beta M(1+\delta)}{M+HCP_t^e}$ the *price expectation multiplier*, which itself is a decreasing function of $P_t^e$.
- It is easy to see that:
  - when $D > 1$, economic agents underpredict price;
  - when $D < 1$, they overpredict price;
  - when $D = 1$, economic agents form correct expectation of price.

# Equilibrium price and output

- Equilibrium price in the model would be that satisfying $D(P^*) = 1$. Indeed, it would guarantee that agents form correct expectations.
- Solving for equilibrium price and output yields:

$$P^* = \frac{M(\beta(1+\delta)-1)}{HC}, \tag{7}$$

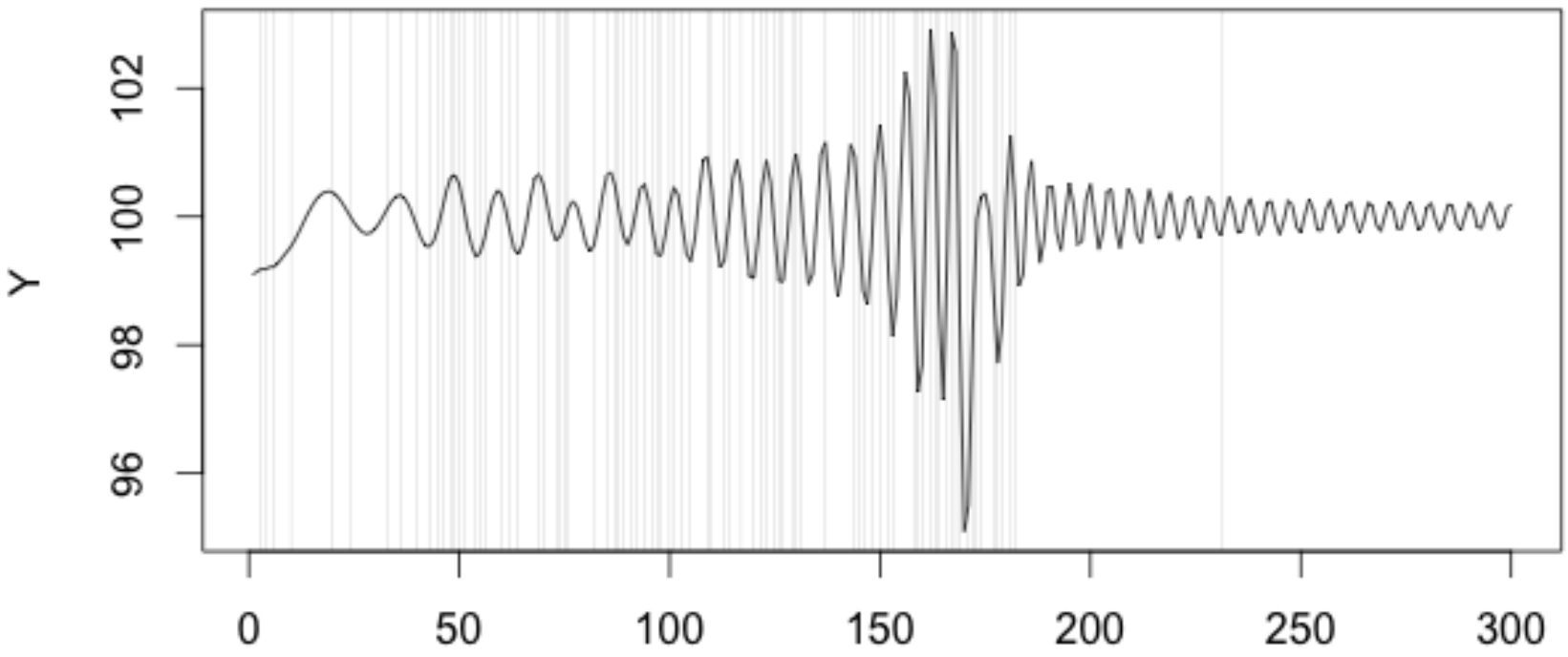
$$Y^* = \frac{HC}{\beta(1+\delta)-1}. \tag{8}$$

- Money is neutral in the long run: any change in money stock would cause exactly the same percentage change in the equilibrium price level, leaving equilibrium output unaffected.

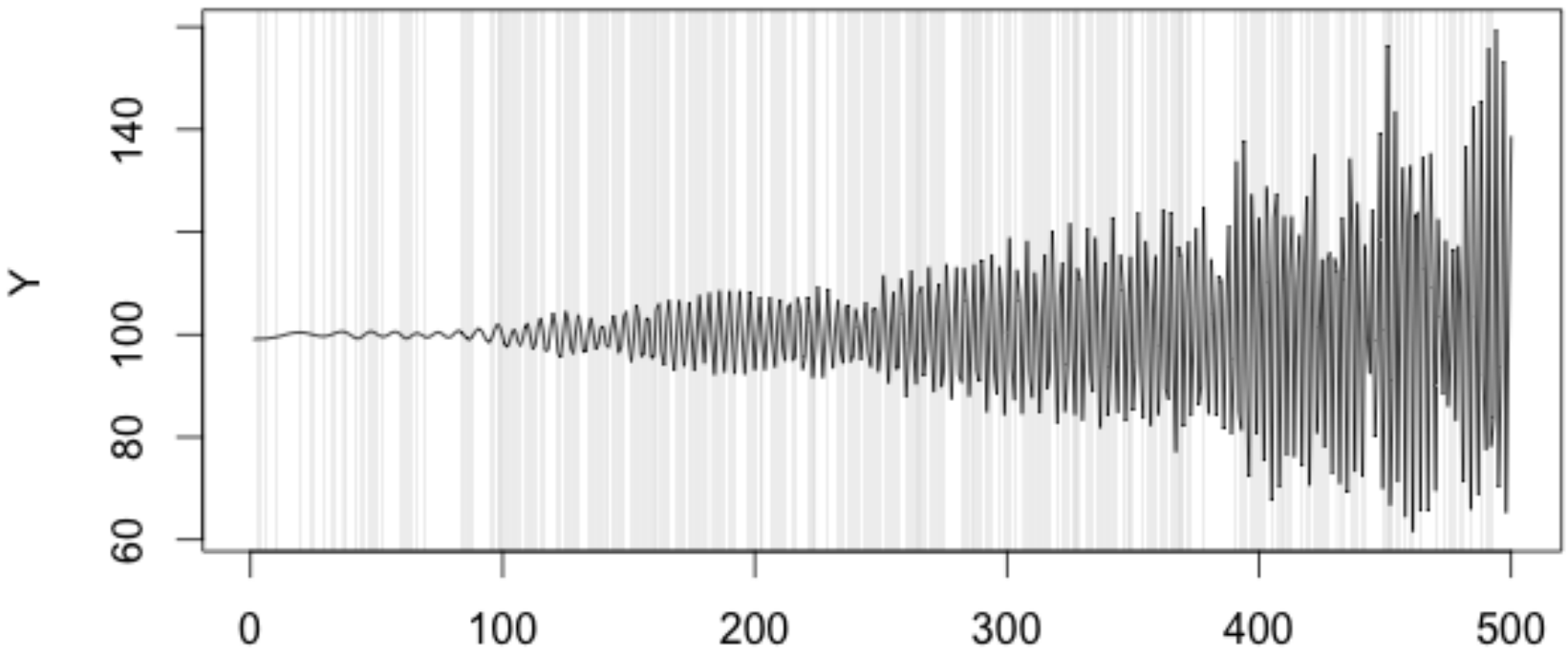
# Possibility of fluctuations

- The possibility of fluctuations driven by adaptive learning and expectations arises from the fact that the ALM function (6) is nonlinear in expected price.
- Economic agents always overestimate (underestimate) future prices when they are higher (lower) than $P^*$.
- Existence of expectation errors makes them learn, i.e. they update their forecasting tools to get more precise predictions.
- As agents approach $P^*$ from either side, they need not necessarily stop in the equilibrium level (which they do not know), and may by inertia enter a zone where the sign of error is opposite; they start to adjust their expectation tool in the opposite direction, and fluctuations arise.

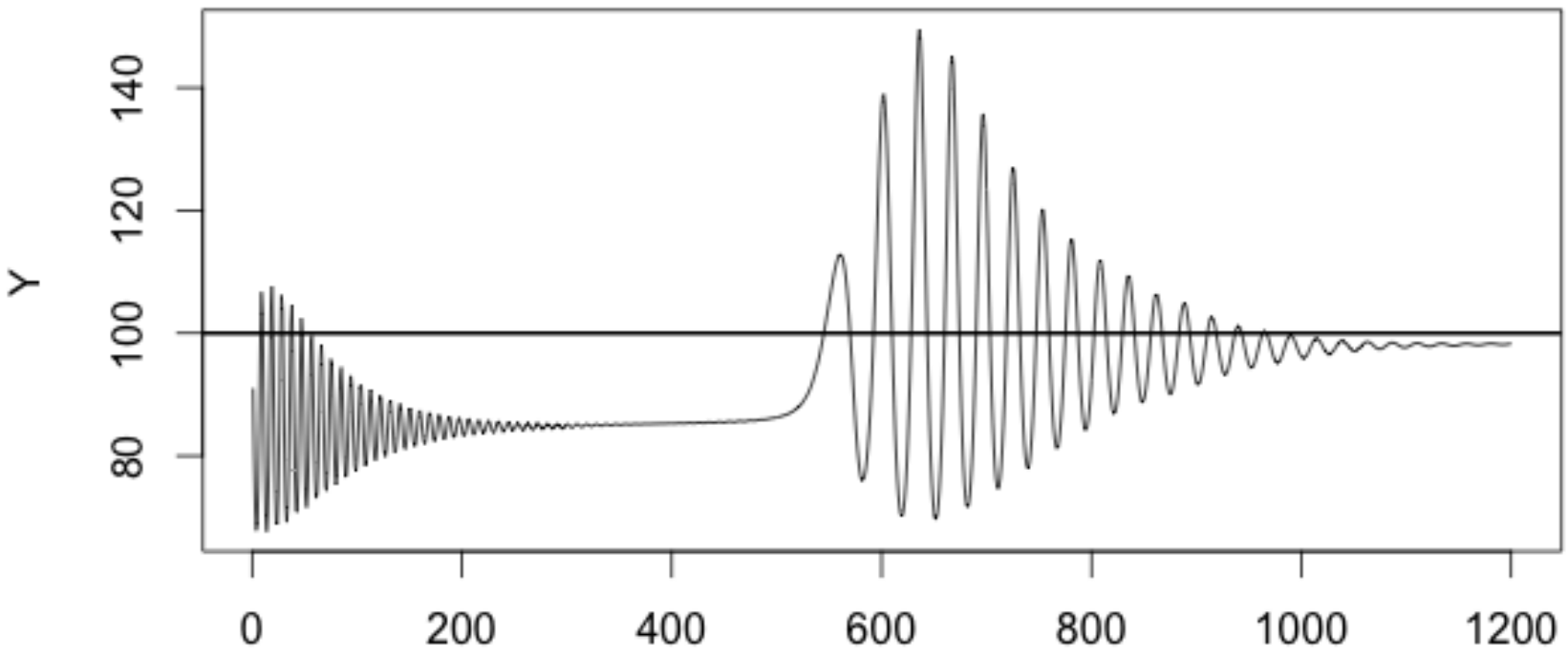
# AutoARIMA learning simulation



# AutoARIMA: diverging cycles



# AR(2): 'false' equilibria



# Insights on e-stability

- Assume agents use an AR(2) model to forecast future prices.
- Q: What is the limiting behaviour of the economy as a function of the parameters? Under what conditions can one observe diverging cycles? -WIP-
- Consider the RLS algorithm:

$$\begin{aligned} \mathbf{\Phi}_t &= \mathbf{\Phi}_{t-1} + t^{-1}\mathbf{R}_t^{-1}\mathbf{p}_{t-1}(P_t - \mathbf{p}_{t-1}^\top \mathbf{\Phi}_{t-1}), \\ \mathbf{R}_t &= \mathbf{R}_{t-1} + t^{-1}(\mathbf{p}_{t-1}\mathbf{p}_{t-1}^\top - \mathbf{R}_{t-1}), \end{aligned} \tag{9}$$

  where $P_t := f(\mathbf{p}_{t-1}, \mathbf{\Phi}_{t-1})$ from the ALM.
- For any given $(\mathbf{\Phi}_0, \mathbf{R}_0, \mathbf{p}_0)$, system (9) describes fully the behaviour of the economy over time.

# Insights on e-stability (cont.)

- Following the literature (e.g. Evans and Honkapohja, 2001), one can work on (9) to obtain the following ODE:

$$\frac{d\mathbf{\Phi}}{d\tau} = \mathbf{\Phi}\left(\frac{\beta M(1+\delta)}{M + HC(\bar{p}^\top \mathbf{\Phi})} - 1\right) \tag{10}$$

- For any starting point $(\mathbf{\Phi}_0, \mathbf{R}_0, \mathbf{p}_0)$, possible limit points of the RLS algorithm correspond to locally stable equilibria of (10).
- By thinking of fixed points in terms of prices (instead of $\mathbf{\Phi}$), one can show that $P^e = P = P^*$ is indeed a fixed point of (10)!
- But is $P^e = P = P^*$ a stable equilibrium of the ODE? Under what conditions? Coming soon...

# Monetary policy in simple model

- Let $M$ be $M_t$. The ALM for total output is:

$$Y_t = \frac{M_t}{\beta(1+\delta)P_t^e} + \frac{HC}{\beta(1+\delta)} \tag{11}$$

- Monetary policy does not have any real effect even in the short run if economic agents know exactly the upcoming change in the money stock *and* adjust their initial price expectation by the same proportion.
- So, monetary policy may have short-term effect if either:
  - it is (in part) unexpected;
  - agents believe that changes in money stock for some reason do not imply exactly the same change in prices.

# Monetary policy in simple model (cont.)

- Let agents believe that $x\%$ change in money stock results in $\psi \cdot x\%$ ($\psi > 0$) change in price level.
- The derivative of actual price by money stock is:

$$\frac{\partial P_t}{\partial M_t} = \beta(1+\delta)\frac{\psi M_t + HCP_t^e}{(M + HCP_t^e)^2} P_t^e, \tag{12}$$

  while it should be $\frac{\partial P_t}{\partial M_t} = \psi \frac{P_t}{M_t} = \beta(1+\delta)\frac{\psi}{M+HCP_t^e} P_t^e$ for agents' expectation to come true.
- If $\psi < 1$ ($\psi > 1$) the 'responsiveness' of actual price is greater (smaller) than $\psi$. Therefore, any value of $\psi$ other than unity is not sustainable from the learning perspective.
- Monetary policy may initially have short-term real effects, but they will disappear in the longer perspective when (and if!) learning is complete.

# Monetary policy: HARA utility

- However, if we use more general HARA functional form for household's utility, ALM of total output becomes:

$$Y_t = \frac{M_t \left(\frac{P_t^e}{P_{t-1}}\right)^{\frac{\gamma}{1-\gamma}} + H^{\frac{1-\gamma}{a}} \left(b_s \left(\frac{P_t^e}{P_{t-1}}\right)^{\frac{\gamma}{1-\gamma}} P_t^e - b_c \delta^{\frac{1}{1-\gamma}} P_{t-1}\right)}{\beta P_t^e \left(\left(\frac{P_t^e}{P_{t-1}}\right)^{\frac{\gamma}{1-\gamma}} + \delta^{\frac{1}{1-\gamma}}\right)} \tag{13}$$

- Even if agents know precisely upcoming change in money stock and adjust their expectation proportionally, monetary policy will still have real effect in the short run.

# What's next

- **Polishing:** bring to perfection what we have so far;
- **Generic analysis:** use generalized functions instead of particular functional forms;
- **Richer models:** build models that would better, yet in stylized and simple way, represent the real world;
- **Calibration:** "a really big step is between simulation and calibration."